مقارنة التوزيعات المتخلفة لتقدير دالة المخاطرة لمرضى أورام السرطان	تواريخ البحث تاريخ تقديم البحث : 2023/7/3 تاريخ قبول البحث :2023/7/24 تاريخ رفع البحث على الموقع: 2024/6/15
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المستخلص :

يهدف البحث الى المقارنة بين توزيع الاسي وتوزيع Akash وتوزيع ليندلي لتقدير دالة المخاطرة لمرضى السرطان الثدي اذ تم استعراض الخصائص الاحصائية للتوزيع الاسي وتوزيع Akash وتوزيع ليندلي وتم تقدير معلماتهم ودالة المخاطرة للتوزيع الاسي وتوزيع Akash وتوزيع ليندلي باستعمال طريقة الامكان الاعظم لغرض الحصول على أفضل النتائج عملت الدراسة على مقارنة بين طرائق التقدير عن طريق تطبيق أسلوب محاكاة مونت كارلو(Carlo Monte) باستعمال برنامج (Wolfram Mathematical 12.2) بحجم عينة (105) وباستعمال ثلاث معايير للمفاضلة بينهما هي $(BIC - AIC - AIC_C)$ في تقدير دالة المخاطرة .

الكلمات المفتاحية: دالة الخطر: التوزيع الأسي، توزيع Akash ، توزيع ليندلي، الحد الأقصى للاحتمال.

Comparing Different Distributions to Estimate the Risk Function for Patients With Cancer Tumors

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Abstract :

The research aims to compare the exponential distribution, the Akash distribution, and the Lindley distribution to estimate the risk function for breast cancer patients. The statistical properties of the exponential distribution, the Akash distribution, the Lindley distribution, and their parameters and the risk function for the exponential distribution, the Akash distribution, and the Lindley distribution were estimated using the method of Likelihood Maximum for the purpose of obtaining the best results. The study is on a comparison between the estimation methods by applying the Monte Carlo simulation method using the program (Wolfram Mathematical 12.2) with a sample size of (105) and using three criteria for comparison between them ($\mathrm{BIC} - \mathrm{AIC} - AIC_C$) in estimating the risk function.

Keywords: Risk Function: Exponential Distribution, Akash Distribution, Lindley Distribution, Likelihood Maximum.

1-The introduction :

Statistical distributions are very important and useful in describing and predicting the real data of the studied phenomenon. In the last decade, researchers have been interested in developing probability distributions and moving them to a comparison between two distributions in order to search for the best representation of the data with the least errors. humans, animals or beings.

2-Research Aim:

It is the study of the exponential distribution, the Akash distribution, the Lindley distribution, the study of its mathematical properties and comparison with each other, as well as the application of these weightings on a sample of real data taken from the cancer tumor center for the deaths of cancer patients, as well as the estimation of the risk function of the proposed distribution.

3-Hazard Rate Function:[5][6]

It represents a conditional probability that the unit (or system) will fail in the period (x, x+Δx), knowing that the machine works (it did not fail) until the time x, and this function is used with multiple names in different scientific fields, so it is called the failure rate in reliability studies, the annihilation force in population studies. The risk function is denoted by h(X) and is expressed in the following formula:

The survival function is defined as a function that is complementary to the cumulative function, as follows:

$$S(x) = Pr[X > x] \rightarrow \; S(x) = 1 - F(X) \qquad (1)$$

$$h(X) = \lim_{\Delta x \to 0} \frac{p(\times < X < x + \Delta x | X > x)}{\Delta x} \qquad (2)$$

$$= \lim_{\Delta x \to 0} \frac{p(\times < X < x + \Delta x)}{p[X > x] * \Delta x}$$

$$h(x) = \frac{1}{S(x)} * \lim_{\Delta x \to 0} \frac{p(\times < X < x + \Delta x | X > x)}{\Delta x} \quad \rightarrow h(x) = \frac{f(x)}{S(x)} \qquad (3)$$

4-Exponential Distribution(ED):[2][3]

The exponential distribution (ED) is one of the continuous distributions, and its name is derived from the exponential function (Function Exponential). stationary. The probability density function of the exponential distribution is a special case of the gamma distribution when substituting β = 1. If the probability density function of the distribution gamma takes the following form:

$$f(x,\beta,\lambda) = \frac{1}{\Gamma\beta \;\; \lambda^{\beta}}\, x^{\beta-1} e^{-\frac{x}{\lambda}}$$

$$f(x,\beta,\lambda) = [\frac{\lambda^{\beta}}{\Gamma\beta}]\, x^{\beta-1} e^{-x\lambda}$$

Substituting β = 1, we get the exponential distribution:

$$f(x\,,\lambda) = f(x) = \lambda e^{-x\lambda} \qquad x\,,\lambda > 0: \qquad (4)$$

Where λ represents the distribution parameter, which is the scale parameter, and is 0 λ>

The cumulative distribution function for a variable that follows the exponential distribution is as follows:

$$F(X) = 1 - e^{-x\lambda} \qquad : \; x\,,\lambda > 0 \qquad (5)$$

And the survival functions s(x) and risk h(x) are in the following form, respectively:

$$s(x) = 1 - F(X) \quad \rightarrow \quad s(x) = 1 - \;\; 1 - e^{-x\lambda} \qquad (6)$$

$$s(x) = \quad e^{-x\lambda} \qquad (7)$$

$$h(x) = \frac{f(x)}{z(x)} = \frac{\lambda e^{-x\lambda}}{e^{-x\lambda}} = \lambda \qquad \dots (8)$$

It is very clear from equation (3) that the risk function for the exponential distribution is equal to the reciprocal of the arithmetic mean of the distribution, and this is what justifies the fact that the function is constant, as:

$$E[X] = \int_0^{\infty} x f(x) dx \qquad \dots (9)$$

$$= \int_0^\infty x\lambda e^{-x\lambda}\, dx = \frac{1}{\lambda}$$

Structural Characteristics of the Exponential Distribution:

(4-11)- Eccentric Moments :(11)

$$\mu_r^* = E(X^r) = \int_0^\infty x^r f(x) dx$$

$$\mu_r^* = E(X^r) = \frac{1}{\lambda} \qquad \dots (10)$$

If r = 1, then it represents the arithmetic mean of the distribution

To find the variance, we use the following formula:

$$v(x) = E(X^2) - \big(E(X)\big)^2 \qquad \dots (11)$$

(4-2)-Central Moments (9)

$$\mu_k = E(x - E(x))^k$$

$$\mu_k = E\left[\sum_{j=0}^{k} C_j^k (-1)^j x^{k-j} \mu_1^{*j}\right] \qquad \dots (12)$$

5-Akash distribution:[10][12][1]

It is one of the continuous distributions presented by Dr. (Rama Shankar) in the year 2015. It consists of a mixture of two components: the exponential distribution has a measurement parameter θ and the gamma distribution has a shape parameter equal to 3 and a measurement parameter θ with certain mixing ratios $\frac{\theta^2}{\theta^2+2}$. And $\frac{2}{\theta^2+2}$respectively, and it has wide applications from real, medical and engineering age data, and it is one of the common and important models used in physical applications, as well as in individual analyzes and error analyzes for various systems and the study of the failure time distribution. The form of the probability and cumulative density function is as follows:

The probability density function of the Akash distribution is a two-component mixture of the exponential distribution having a scale parameter θ and a Kama distribution having a shape parameter equal to 3 and a scale parameter θ with given mixing ratios $\frac{\theta^2}{\theta^2+2}$ and $\frac{2}{\theta^2+2}$ respectively take the following form:

$$f^*(x) = \frac{\theta^3}{\theta^2+2}(1+x^2)e^{-\theta x} \qquad ; x>0\ ,\theta>0 \qquad ...(13)$$

The cumulative distribution function for a variable that follows the Akash distribution is as follows:

$$F^*(x) = 1 - \left[1 + \frac{\theta x(\theta x+2)}{\theta^2+2}\right] e^{-\theta x} \qquad ; x > 0\ , \theta > 0 \qquad ...(14)$$

And the survival functions s(x) and risk h(x) are in the following form, respectively:

$$S(x) = 1 - F(x\ ) \rightarrow s(x) = 1 + \frac{\theta x(\theta x+2)}{\theta^2+2} e^{-\theta x} \qquad ...\ (15)$$

$$h(x) = \frac{f(x)}{S(x)}$$

$$h(x) = \frac{\frac{\theta^3}{\theta^2+2}(1+x^2)e^{-\theta x}}{1 - \frac{\theta x(\theta x+2)}{\theta^2+2}e^{-\theta x}} \qquad (16)$$

Structural Characteristics of Akash Distribution:

(5-1)-eccentric moments; [11]

$$E[X] = \int_0^\infty x f^*(x) dx \qquad ...(17)$$

$$E[X] = \int_0^\infty x[\frac{\theta^3}{\theta^2+2}(1+x^2)]e^{-\theta x} dx \qquad ...(18)$$

$$\mu_r^* = E(X^r) = \int_0^\infty x^r[\frac{\theta^3}{\theta^2+2}(1+x^{r2})]e^{-\theta x^r} dx \qquad (19)$$

If r = 1, then it represents the arithmetic mean of the distribution

To find the variance, we use the following formula:

$$v(x) = E(X^2) - \left(E(X)\right)^2 \qquad \dots (20)$$

(5-2)-central moments :[9]

$$\mu_k = E(x - E(x))^k$$

$$\mu_k = E\left[\sum_{j=0}^{k} C_j^k (-1)^j x^{k-j} \mu_1^{*j}\right] \qquad \dots (21)$$

6- Lindly Distribution:[7]

The Lindley distribution is one of the continuous distributions resulting from mixing two random variables, one of which follows the exponential distribution with a measurement parameter (α), which is characterized by a great potential in representing different systems that are damaged by complex and heterogeneous societies, as well as the high flexibility of this distribution as a failure model. The other one follows the gamma distribution with two parameters (α) and shape.

The probability density function of the Lindley distribution is:

$$k(x;\theta) = \frac{\alpha^2}{(1+\alpha)} (1+x)e^{-\alpha x} \qquad , x < 0, \alpha > 0 \qquad (22)$$

The cumulative distribution function for a variable that follows the Lindley distribution is as follows:

$$K(x;\alpha) = 1 - \left\{1 + \frac{\alpha}{(1+\alpha)} X\right] e^{-\alpha x} \qquad , x > 0, \alpha > 0 \qquad (23)$$

And the survival functions s(x) and risk h(x) are in the following form, respectively:

$$S(x) = 1 - K(x) = \left[1 + \frac{\alpha X}{1+\alpha}\right] e^{-\alpha x} \qquad (24)$$

$$h(x) = \frac{k(x)}{S(x)}$$

$$h(x) = \frac{\alpha^2(1+X)}{1+\alpha(1+X)} \qquad (25)$$

(6-1)-eccentric moments; [11]

$$E[X] = \int_0^\infty x^r k(x)dx \qquad \dots(26)$$

$$E[X] = \int_0^\infty x[\frac{\alpha^2}{(1+\alpha)}(1+x)e^{-\alpha x}]dx \qquad \dots(27)$$

$$\mu_r^* = E(X^r) = \int_0^\infty x^r \left[\frac{\alpha^2}{(1+\alpha)}(1+x^r)e^{-\alpha x^r}\right]dx \qquad (28)$$

If r = 1, then it represents the arithmetic mean of the distribution

To find the variance, we use the following formula:

$$v(x) = E(X^2) - \left(E(X)\right)^2 \qquad \dots(29)$$

(6-2)-central moments :[9]

$$\mu_k = E(x - E(x))^k$$

$$\mu_k = E\left[\sum_{j=0}^{k} C_j^k (-1)^j x^{k-j} \mu_1^{*j}\right] \qquad \dots(30)$$

7- Estimation methods

(7-1) - Estimating the method of Likelihood Maximum of the exponential distribution :[4][7]

This method was proposed by R.A.Fisher in the year (1920) and is characterized by important estimation methods due to the accuracy of its estimates, as this method is characterized by efficiency, sufficiency, stability and consistency in addition to impartiality.

$$\iota f(x,\lambda) = \prod_{i=1}^{n} \lambda e^{-x\lambda} \quad = \lambda^n e^{-\sum x_i \lambda} \qquad \dots(31)$$

$$lnlf(x,\lambda) = \sum_{i=1}^{n} Ln\, Lf(x,\lambda) = nln(\lambda) - \lambda \sum x_i$$

$$\frac{\partial lnlf(x,\lambda)}{\partial \lambda} = \frac{n}{\hat{\lambda}} - \sum x_i$$

$$\frac{n}{\hat{\lambda}} - \sum x_i = 0 \rightarrow \frac{n}{\hat{\lambda}} = \sum x_i \rightarrow \hat{\lambda} \sum x_i = n \rightarrow \hat{\lambda} = \frac{n}{\sum x_i} = \frac{1}{\bar{x}} \qquad (32)$$

The equations represent a system of nonlinear equations that cannot be solved except by using one of the numerical methods. In order to obtain the estimators, the Newton-Raphsen method has been used.

(7-2)- Estimating the method of Maximum Likelihood of Akash distribution: [7][4]

This method was proposed by R.A.Fisher in the year (1920) and is characterized by important estimation methods due to the accuracy of its estimates, as this method is characterized by efficiency, sufficiency, stability and consistency in addition to impartiality.

Suppose $(x_1, x_2, \ldots x_n)$ is an independent random sample of size n from the Akash distribution with parameters θ and , ln L, can be obtained by

$$lf^*(x) = \prod_{i=1}^{n} [\frac{\theta^3}{\theta^2+2}(1+x^2)e^{-\theta x}] \quad ....(33)$$

$$lnlf^*(x) = \sum_{i=1}^{n} lnlf^*(x)$$

$$\ln l = \left[\begin{array}{c} 3n \ln\theta - n\ln(\theta^2+2) + \\ \sum_{i=1}^{n} \ln(1+x_i{}^2) \\ -\theta \sum_{i=1}^{n} x_i \end{array} \right]$$

$$\frac{dlnlf^*(x)}{d\theta} = \frac{n(6+\hat{\theta}^2)}{\theta(2+\hat{\theta}^2)} \sum_{i=1}^{n} x_i \quad ...(34)$$

$$\frac{\mathrm{dlnlf}^*(x)}{d\theta}=\frac{n\left(6+\hat{\theta}^2\right)}{\theta\left(2+\hat{\theta}^2\right)}-\sum_{i=1}^{n}x_i=\theta$$

$$=\frac{1}{3\sum_{i=1}^{n}x_i}\left(n+\frac{-n^2+6(\sum_{i=1}^{n}x_i)^2}{\left(-n^2-72n(\sum_{i=1}^{n}x_i)^2+3\sqrt{6}\sqrt{3n^4(\sum_{i=1}^{n}x_i)^2+94n^2(\sum_{i=1}^{n}x_i)^4+4(\sum_{i=1}^{n}x_i)^6}\right)^{\frac{1}{3}}}-\left(-n^2-72n\left(\sum_{i=1}^{n}x_i\right)^2+3\sqrt{6}\sqrt{3n^4\left(\sum_{i=1}^{n}x_i\right)^2+94n^2\left(\sum_{i=1}^{n}x_i\right)^4+4\left(\sum_{i=1}^{n}x_i\right)^6}\right)^{\frac{1}{3}}\right) \quad (35)$$

We can obtain estimates of the unknown parameters of equation (35) by settling them to zero and solving one at a time. These equations are an implicit form, so they can be solved using numerical iteration such as the Newton-Raphson algorithm.

(3-7)- Estimating the Maximum Likelihood method for the Akash distribution: [7][4]

This method was proposed by R.A.Fisher in the year (1920) and is characterized by important estimation methods due to the accuracy of its estimates, as this method is characterized by efficiency, sufficiency, stability and consistency in addition to impartiality, if $(x_1, x_2, \dots x_n)$ represents an independent random sample of the observations of a variable that follows the Lendl distribution with a measurement parameter (α), then the possibility function for the observations of the random variable is:

$$l(\alpha, x_1, x_2, \dots x_n)=\frac{\alpha^{2n}}{(1+\alpha)^n}\prod_{i=1}^{n}(1+x_i)e^{-\alpha\sum_{i=1}^{n}x_i} \qquad (36)$$

$$lnl(\alpha, x_1, x_2, \dots x_n)=2n\ln\alpha-n\ln(1+\alpha)+ln\prod_{i=1}^{n}(1+x_i)-\alpha\sum_{i=1}^{n}x_i$$

By partially deriving both sides of equation (36) with respect to parameter (α):

$$\frac{\mathrm{d}lnl(\alpha, x_1, x_2, \dots x_n)}{\mathrm{d}\alpha}=\frac{2n}{\alpha}-\frac{n}{(1+\alpha)}-\sum_{i=1}^{n}x_i$$

$$=\frac{2n(\alpha+1)-n\alpha-\alpha(\alpha+1)\sum_{i=1}^{n}x_i}{\alpha(\alpha+1)} \qquad (37)$$

By equating equation (37) to zero and dividing by n:

$$2(1+\hat{\alpha})-\hat{\alpha}-\hat{\alpha}(1+\hat{\alpha})\bar{x}=0$$

$$\bar{x}\hat{\alpha}^2 + (\bar{x} - 1)\hat{\alpha} - 2 = 0 \qquad (38)$$

By solving equation (9-7), we obtain the maximum possibility method estimator for parameter (α):

$$\hat{\alpha}_{ML} = \frac{-(1-\bar{X}) + \sqrt{(\bar{X}-1)^2 + 8\bar{X}}}{2\bar{X}} \qquad ,\bar{X} > 0 \qquad (39)$$

8- Criteria for selection of the best distribution:[10]

The process of choosing the best distribution is an important process in analyzing the data and to prove the preference of the exponential distribution or the Akash distribution and its suitability for the real data of cancer. Three comparison criteria were used, which are:

(8-1) -Akanke information criteria**(AIC)**:-

The Akaike criterion was proposed by the researcher (1973, Akaike Petrov and Csaki) (the logarithm of the function of greatest possibility is used, and then the AIC is calculated for each distribution, and the distribution that has the lowest value of the criterion is the best distribution, its general formula is as follows:

$$AIC = -2\log(l) + 2r \qquad (40)$$

Since: log(l): the logarithm of the maximum possibility function for the sample observations.

r : number of parameters of the distribution.

(8-2) Correction Akaike information criteria $(\boldsymbol{AIC_C})$is a criterion for selecting the best distribution based on $(\boldsymbol{AIC_C})$and its formula is as follows:

$$\boldsymbol{AIC_c = AIC + \frac{2r(r+1)}{n-r-1}} \qquad \ldots(\mathbf{41})$$

As: AIC: IKEA standard

r: the number of parameters of the distribution.

n: sample size.

(8-3)-(Bayesian Information Criterion(BIC):

$$BIC = M\,Ln(n) - 2Ln(L) \qquad \ldots(42)$$

Also, the distribution that has the lowest value for this criterion will be the best distribution in representing the data.

9-Application side:

This section includes a practical application of the Exponential distribution, the Akash distribution, and the Lindley distribution on real-world data represented by survival times until death for the registered cases of patients with cancer of the cancer tumor center in Basra Governorate for the period since the opening of the tumor center in Basra Governorate in 2009 until October of 2009 2020 and an estimate of the risk function for each patient in order to draw a picture of the possibility of sudden death for patients with this type of disease, where the simulation was implemented using the Mathematic-13 program on a sample size (105) to find out the effect of the sample size on the results of the estimation methods, and the experiment was repeated 1000 once for each model to obtain the highest possible homogeneity.

(9-1)- Description of the data:

Real data were collected from the register of patients registered in the Oncology Center in Basra Teaching Hospital in Basra Governorate, who were diagnosed with jaw cancer and died. (per week) and include them in Table (1) as follows:

Duration of survival of a patient with breast cancer from the date of admission to the hospital until death (in weeks)

i	xi	i	xi	i	xi	i	Xi
1	0.02	29	0.94	57	2.21	83	**3.45**
2	0.04	30	0.98	58	2.22	84	**3.53**
3	0.04	31	1.00	59	2.24	85	**3.54**
4	0.06	32	1.03	60	2.25	86	**3.59**
5	0.11	33	1.04	61	2.30	87	**3.61**
6	0.15	34	1.18	62	2.37	88	**3.65**
7	0.15	35	1.19	63	2.38	89	**3.94**
8	0.19	36	1.21	64	2.42	90	**3.95**
9	0.23	37	1.23	65	2.44	91	**3.97**
10	0.28	38	1.23	66	2.46	92	**4.00**
11	0.29	39	1.25	67	2.47	93	**4.03**
12	0.32	40	1.29	68	2.50	94	**4.31**
13	0.39	41	1.31	69	2.56	95	**4.34**
14	0.46	42	1.34	70	2.57	96	**4.40**
15	0.46	43	1.39	71	2.58	97	**4.66**
16	0.48	44	1.45	72	2.61	98	**4.96**
17	0.50	45	1.51	73	2.67	99	**4.97**
18	0.51	46	1.62	74	2.69	100	**5.39**
19	0.58	47	1.66	75	2.74	101	**5.70**
20	0.59	48	1.72	76	2.78	102	**5.80**
21	0.62	49	1.82	77	2.81	103	**7.44**
22	0.79	50	1.94	78	2.86	104	**7.94**
23	0.83	51	2.01	79	2.90	105	**8.78**
		24	0.83	52	2.02	80	**3.13**
		25	0.83	53	2.05	81	**3.19**
		26	0.85	54	2.11	82	**3.35**
		27	0.91	55	2.11	57	**2.21**
		28	0.93	56	2.15	58	**2.22**

The following table shows the most important statistics of the real data sample

Schedule (2)

Shows the most important primary descriptive statistics of the real sample data

mean	**2.20767**
Variance	**2.99713**
skewness	**1.25652**
kurtosis	**5.01351**
median	**2.05**
Standard Deviation	**1.73122**

(9-2)Data analysis:

The real data sample was analyzed using the greatest possibility method. Table (3) shows the estimates of the parameters for the proposed (Exponential and Akash distribution) and comparison criteria between the distributions:

Table (3)

Shows parameter estimates for the proposed (Exponential, Akash, and l) distribution and comparison criteria for the real data.

parameter				AIC	AIC_C	BIC
dist	θ	α	λ			
AK	0.452966	-	-	372.155	369.54	369.501
Exp	-	-	1.04066	380.961	378.346	378.307
Lind		0.716808		374.482	371.867	371.828

By testing the following hypotheses and according to the criteria mentioned, it was found that:

H_0: (Akash) Distribution tracking data

H_1: (Akash) Distribution Tracking No DataAkash

The estimated parameter values of the A (Akash) distribution were in agreement with the default values of the parameters shown alongside the simulation.

The preference of the (Akash) distribution as a result of having less (BIC،AIC, $AIC_{C,}$ criteria) and thus is the best distribution in representing and describing the sample under study.

After analyzing the data, the values and risk function were extracted for the exponential distribution and the Akash distribution, and the results are explained in the following table:

Table (4)

An estimator and risk function for the exponential distribution and the Akash distribution for real data

h(t)(Exponential)	h(t)(Akash)	xi	i	h(t)(Exponential)	h(t)(Akash)	Xi	i
0.452966	0.360275	0.02	1	0.452966	0.487723	2.01	**51**
0.452966	0.357168	0.04	2	0.452966	0.488718	2.02	**52**
0.452966	0.356564	0.04	3	0.452966	0.492678	2.05	**53**
0.452966	0.352503	0.06	4	0.452966	0.499526	2.11	**54**
0.452966	0.343791	0.11	5	0.452966	0.500173	2.11	**55**
0.452966	0.336997	0.15	6	0.452966	0.504355	2.15	**56**
0.452966	0.336997	0.15	7	0.452966	0.510708	2.21	**57**
0.452966	0.331469	0.19	8	0.452966	0.512281	2.22	**58**
0.452966	0.327844	0.23	9	0.452966	0.514786	2.24	**59**
0.452966	0.32437	0.28	10	0.452966	0.515721	2.25	**60**
0.452966	0.323173	0.29	11	0.452966	0.521592	2.30	**61**
0.452966	0.321905	0.32	12	0.452966	0.528579	2.37	**62**
0.452966	0.319874	0.39	13	0.452966	0.529781	2.38	**63**
0.452966	0.319802	0.46	14	0.452966	0.534252	2.42	**64**
0.452966	0.319836	0.46	15	0.452966	0.536614	2.44	**65**
0.452966	0.320148	0.48	16	0.452966	0.538375	2.46	**66**
0.452966	0.320585	0.50	17	0.452966	0.539544	2.47	**67**
0.452966	0.320809	0.51	18	0.452966	0.54245	2.50	**68**
0.452966	0.32362	0.58	19	0.452966	0.54847	2.56	**69**
0.452966	0.324325	0.59	20	0.452966	0.549888	2.57	**70**
0.452966	0.325882	0.62	21	0.452966	0.550453	2.58	**71**
0.452966	0.33841	0.79	22	0.452966	0.554104	2.61	**72**
0.452966	0.342069	0.83	23	0.452966	0.55964	2.67	**73**
0.452966	0.342339	0.83	24	0.452966	0.562372	2.69	**74**
0.452966	0.342609	0.83	25	0.452966	0.566694	2.74	**75**
0.452966	0.344533	0.85	26	0.452966	0.570425	2.78	**76**
0.452966	0.350286	0.91	27	0.452966	0.573586	2.81	**77**
0.452966	0.352684	0.93	28	0.452966	0.577749	2.86	**78**
0.452966	0.354515	0.94	29	0.452966	0.581598	2.90	**79**
0.452966	0.358572	0.98	30	0.452966	0.601983	3.13	**80**
0.452966	0.360803	1.00	31	0.452966	0.607367	3.19	**81**
0.452966	0.364695	1.03	32	0.452966	0.620239	3.35	**82**
0.452966	0.365352	1.04	33	0.452966	0.628036	3.45	**83**
0.452966	0.381992	1.18	34	0.452966	0.633928	3.53	**84**
0.452966	0.384089	1.19	35	0.452966	0.634965	3.54	**85**
0.452966	0.386899	1.21	36	0.452966	0.638456	3.59	**86**

0.452966	0.388309	1.23	37	0.452966	0.639878	3.61	**87**
0.452966	0.388663	1.23	38	0.452966	0.642896	3.65	**88**
0.452966	0.391496	1.25	39	0.452966	0.662761	3.94	**89**
0.452966	0.396483	1.29	40	0.452966	0.663307	3.95	**90**
0.452966	0.399346	1.31	41	0.452966	0.664577	3.97	**91**
0.452966	0.402936	1.34	42	0.452966	0.666737	4.00	**92**
0.452966	0.409779	1.39	43	0.452966	0.668874	4.03	**93**
0.452966	0.417359	1.45	44	0.452966	0.68602	4.31	**94**
0.452966	0.424576	1.51	45	0.452966	0.687477	4.34	**95**
0.452966	0.439655	1.62	46	0.452966	0.690996	4.40	**96**
0.452966	0.444642	1.66	47	0.452966	0.705474	4.66	**97**
0.452966	0.451721	1.72	48	0.452966	0.720501	4.96	**98**
0.452966	0.46465	1.82	49	0.452966	0.72077	4.97	**99**
0.452966	0.479691	1.94	50	0.452966	0.739916	5.39	**100**
				0.452966	0.752991	5.70	**101**
				0.452966	0.756684	5.80	**102**
				0.452966	0.808955	7.44	**103**
				0.452966	0.821203	7.94	**104**
				0.452966	0.839325	8.78	**105**
				47.56143	51.634241	231.81	**sum**
				0.452966	0.4917547	2.207714	**mean**

From Table (9) it is clear that the values of the estimated risk function are increasing with the increase in the patient's survival time and tend to be stable when its values approach the value of the parameter θ which is (0.452966), and this confirms what was proposed in the theoretical aspect of it This study, as we note that the values of The risk function is increasing until the patient reaches a survival period of approximately 18 months, then it begins to stabilize when this period is exceeded, and this explains the true behavior of the disease, as when this period is exceeded, the patient is closer to recovery than to death.

10- Conclusions and Recommendations:

Based on the results of the applied side, the researcher reached the following:

(10-1) - Conclusions: Single distributions such as the Akash distribution represent such data, and using them gives flexibility in the estimation process, as knowing the behavior of the phenomenon can be predicted more accurately than using other distributions. Survival is decreasing with time, and the risk function is an increasing function with time, and we conclude from that that the greater the time of infection, the greater the probability of death of a person due to this disease.

(10-2) Recommendations: Using Bayesian methods to estimate the risk function of the (Exponential) distribution and conducting a comparative study with the classical methods used in this study or other methods.

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